**Portfolio Active Return Attribution – Univariate Regression Based**

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**1. Definition of notations**

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| **Notation** | **Description** |
|  | Active weight of stock i () |
|  | Return of stock i (after de-mean) |
|  | Return of active portfolio () |
|  | Weight of factor k () |
|  | Return of factor k, estimated from cross-sectional univariate regression |
|  | Return of alpha, estimated from cross-sectional univariate regression |
|  | Normalized factor exposure of stock i to factor k, it has zero mean and constant cross-sectional variance |
|  | Final Alpha of stock i () |
|  | Unexplained return of stock i to factor k, estimated from cross-sectional univariate regression |
|  | Unexplained return of stock i to alpha |

**2. Stock Return Attribution**

Both stock returns and alpha have zero mean, so following cross sectional regression can be fitted to find and

At the same time, the same regression technique can be used to find factor return

If the above linear regression is fitted in an OLS fashion, then we know and are in fact returns of signal weighted portfolios. Given that , it leads to that:

, where is a constant whose value is determined by the activeness of the factor portfolio of factor k.

**3. Portfolio Active Return Attribution**

Above all we have:

, where is the active alpha exposure of the portfolio, and is active portfolio weighted specific return.

In this way we decompose portfolio active return into two parts: return from alphas (which are systematic) and return from stock specifics (which are idiosyncratic).